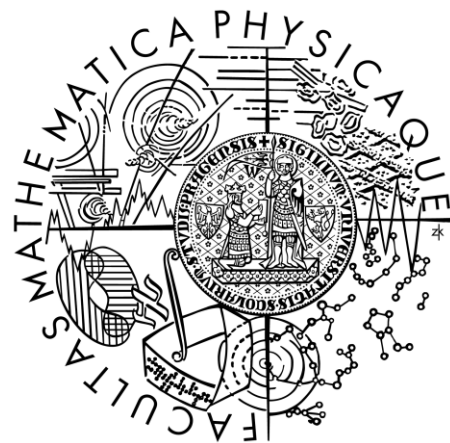


Implementation of Sprouts: a graph drawing game



Tomáš Čížek and Martin Balko
Charles University, Prague

Rules

A two-player game with simple rules:

- Starts with n initial spots.
- Players alternate in connecting spots by curves.
- The curves cannot cross.
- A new spot is added along a newly drawn curve.
- Each spot can be incident to at most three curves.
- The first player who cannot make a move loses.

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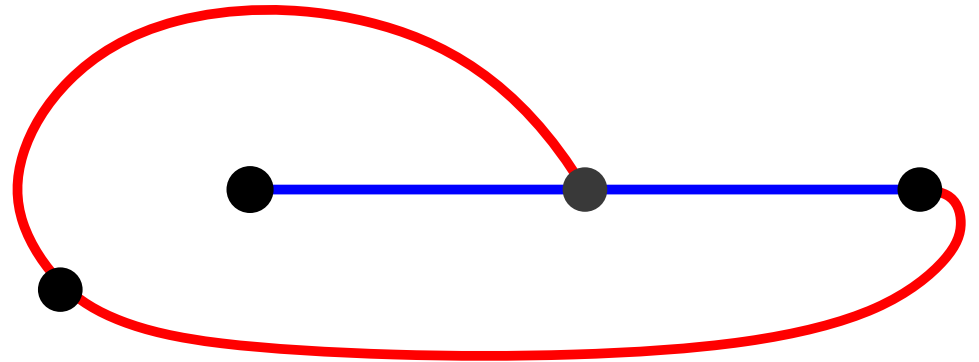
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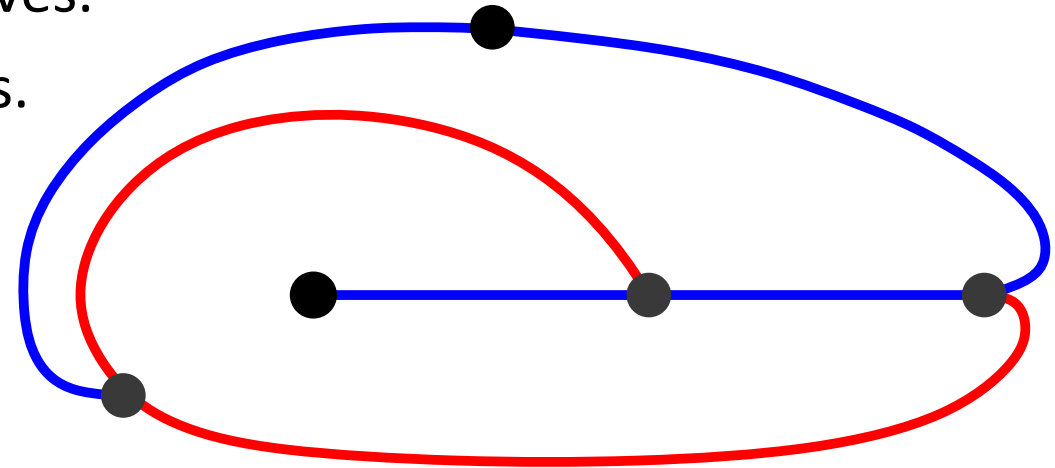
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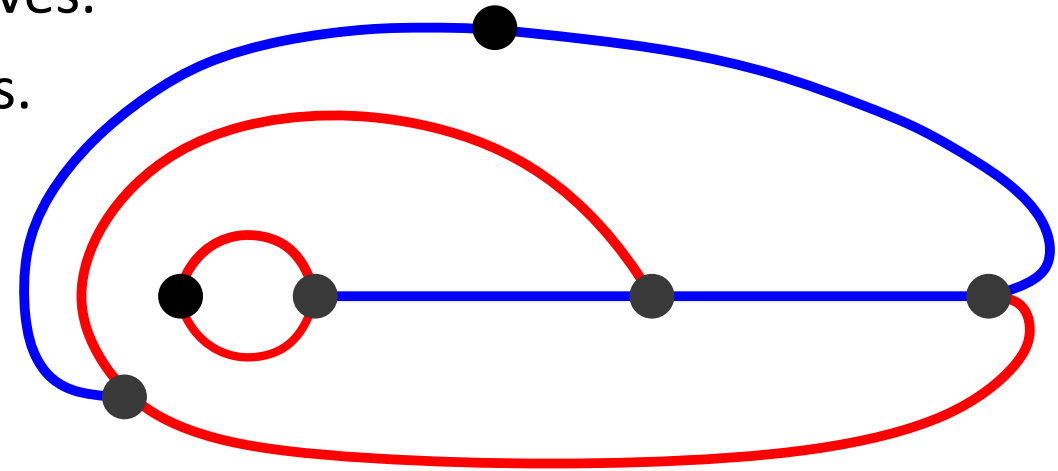
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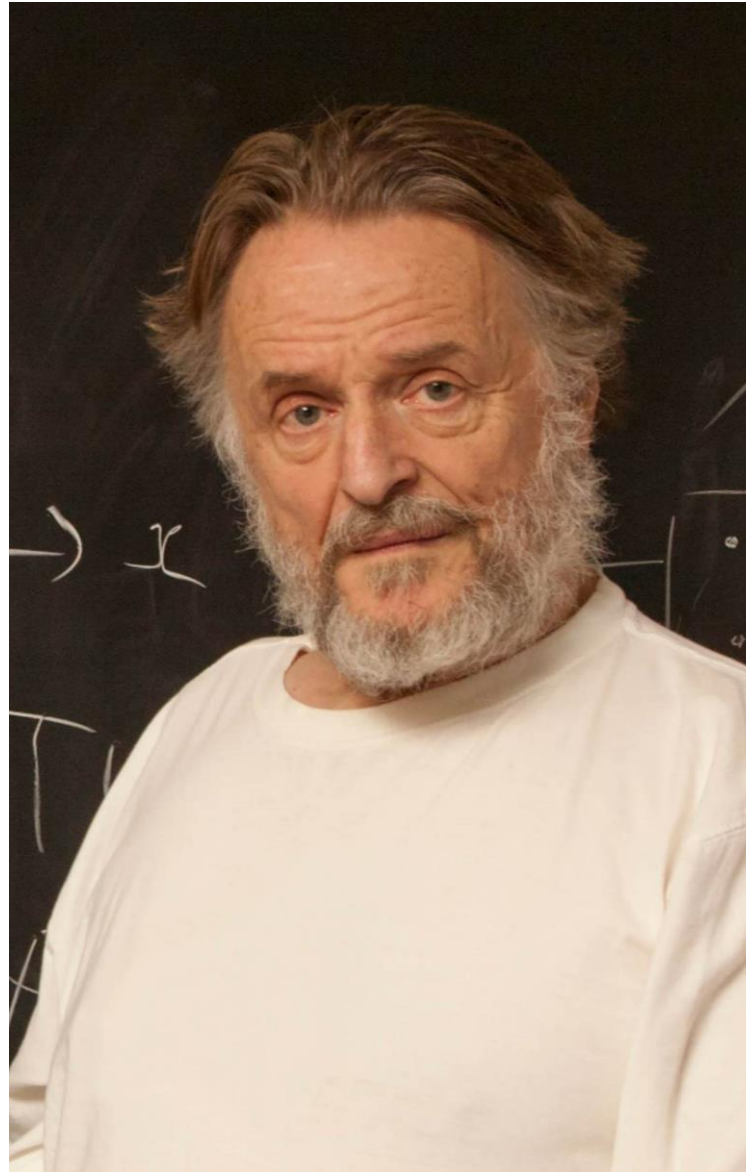
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Background

- Designed by British mathematicians [J. Conway](#) and [M. Paterson](#) in 1967 as a game that would resist computer analysis.
- Dozens of papers analyzing Sprouts were published.
- However, no solid Sprouts application has existed for over [half a century](#).
- Until now...



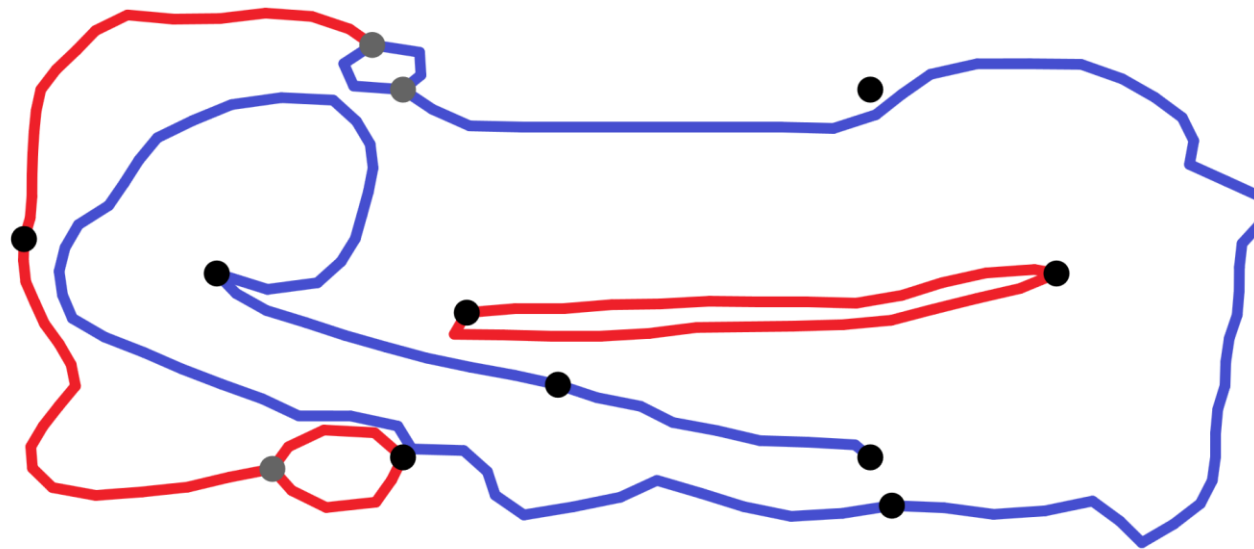
www.princeton.edu



www.alchetron.com/Mike-Paterson

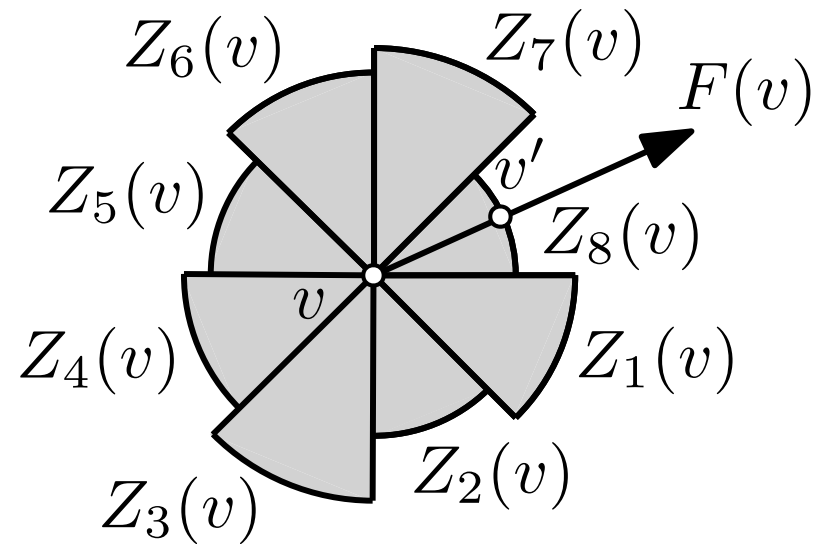
1st barrier – positions degenerate

- Free form input drawings make positions confusing over time.
- Therefore, we need a mechanism that would maintain positions clear.



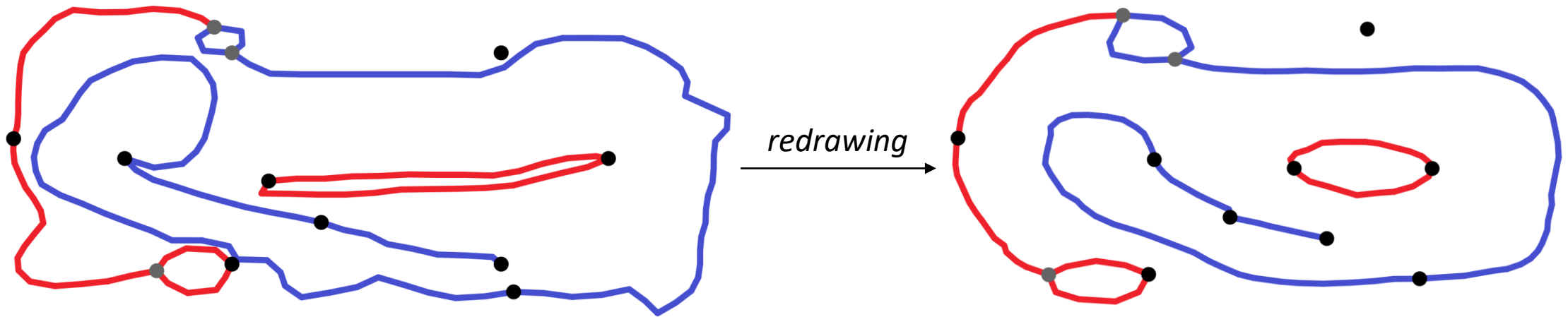
Redrawing algorithm

- We modified the force-directed algorithm *ImPrEd* [Simonetto et al., 2011].
- The algorithm iteratively computes a force for each vertex. Each vertex is then moved in the direction of its force so that no crossing is created.



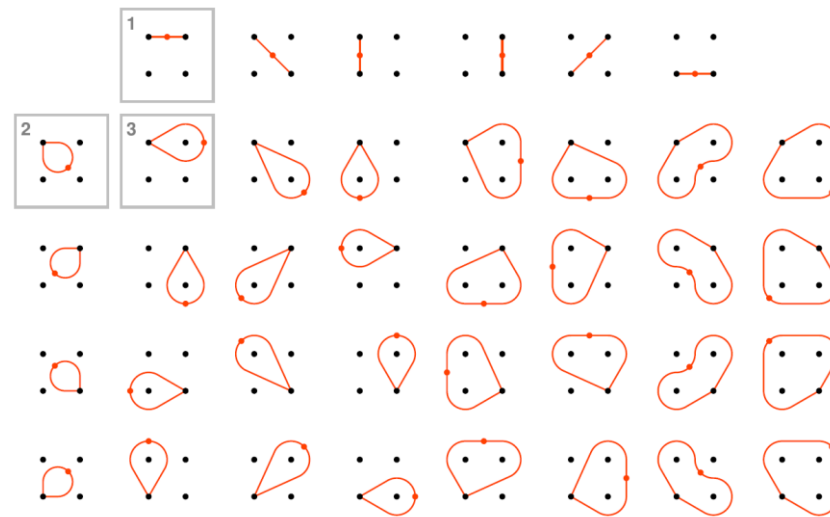
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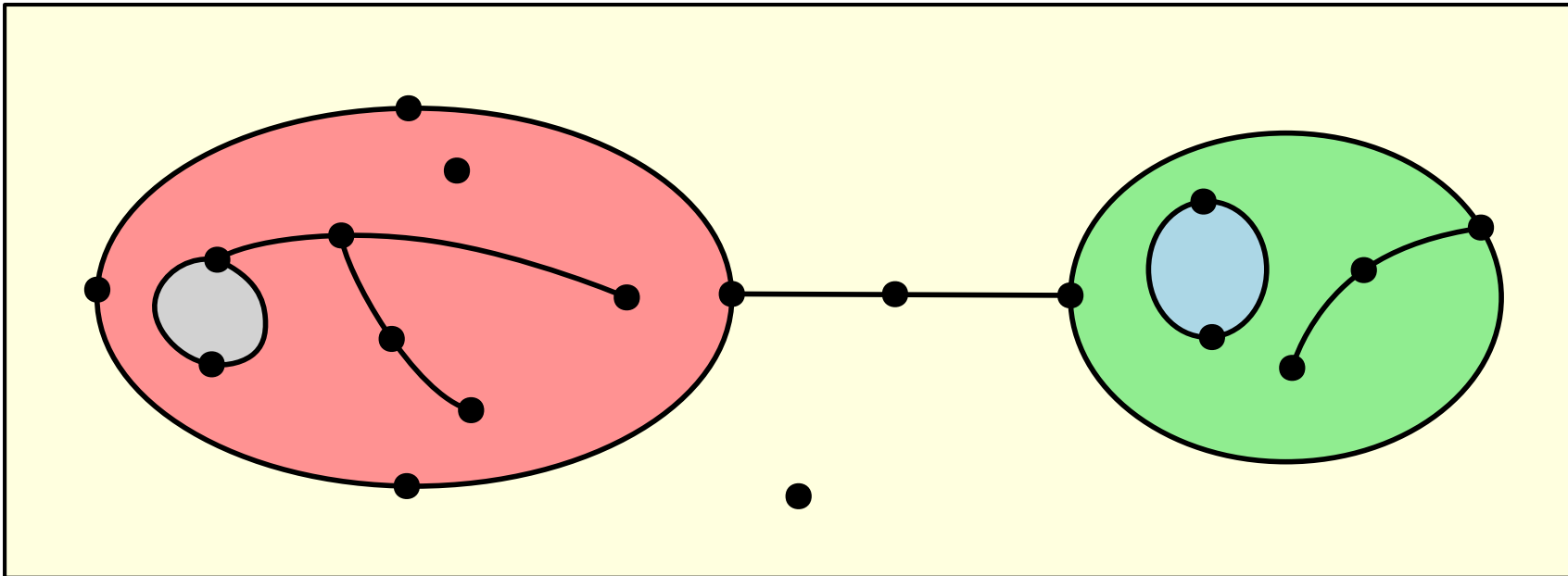
2nd barrier – enormous game trees

- The number of moves can **grow exponentially** in the number of initial spots.
- Thus, we need to include many optimizations of the game tree exploration.
- Luckily, some of them have been already introduced (a Sprouts solver **GLOP** [Lemoine and Viennot, 2015]).



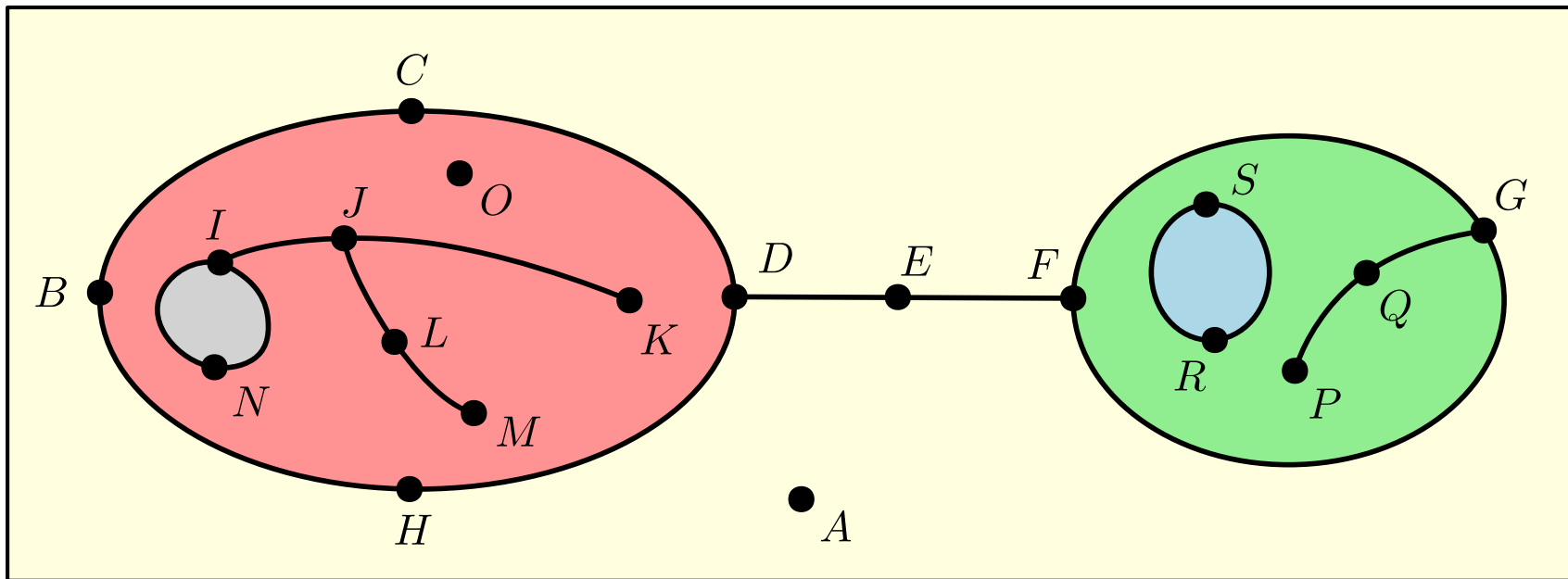
Positions encoding

- We need to **encode game properties** of a position as unambiguously as possible to be able to explore its game tree.



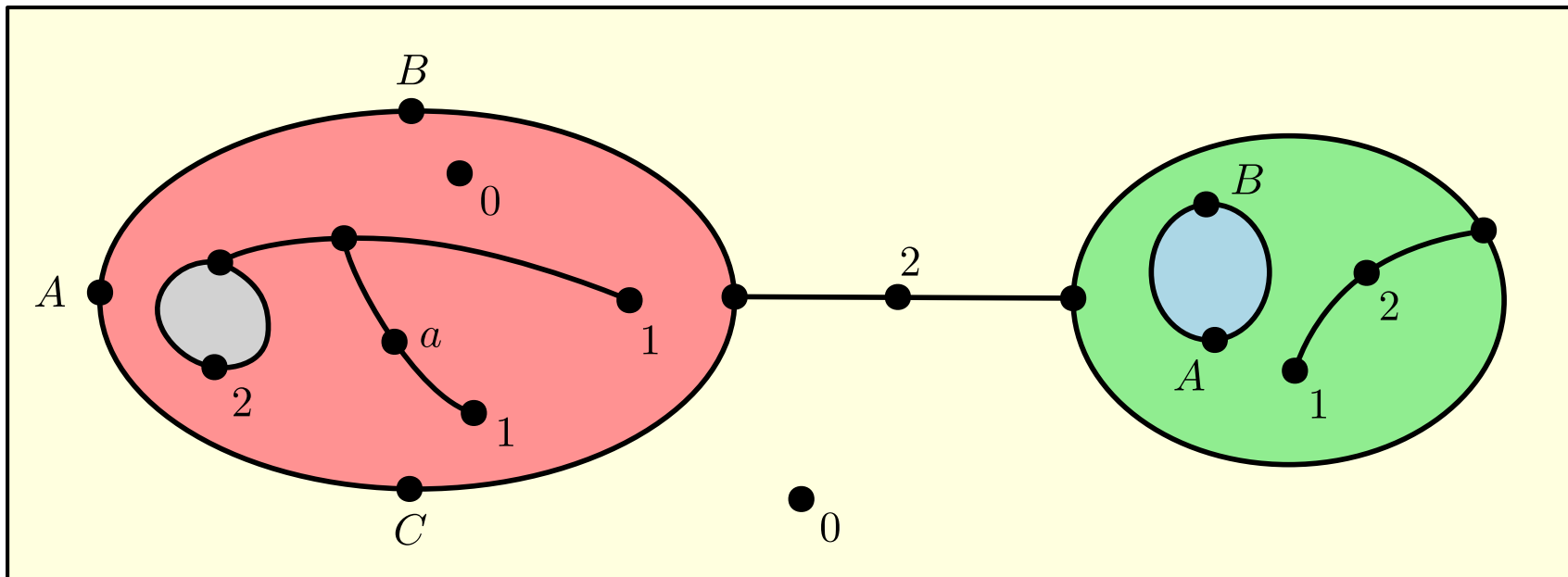
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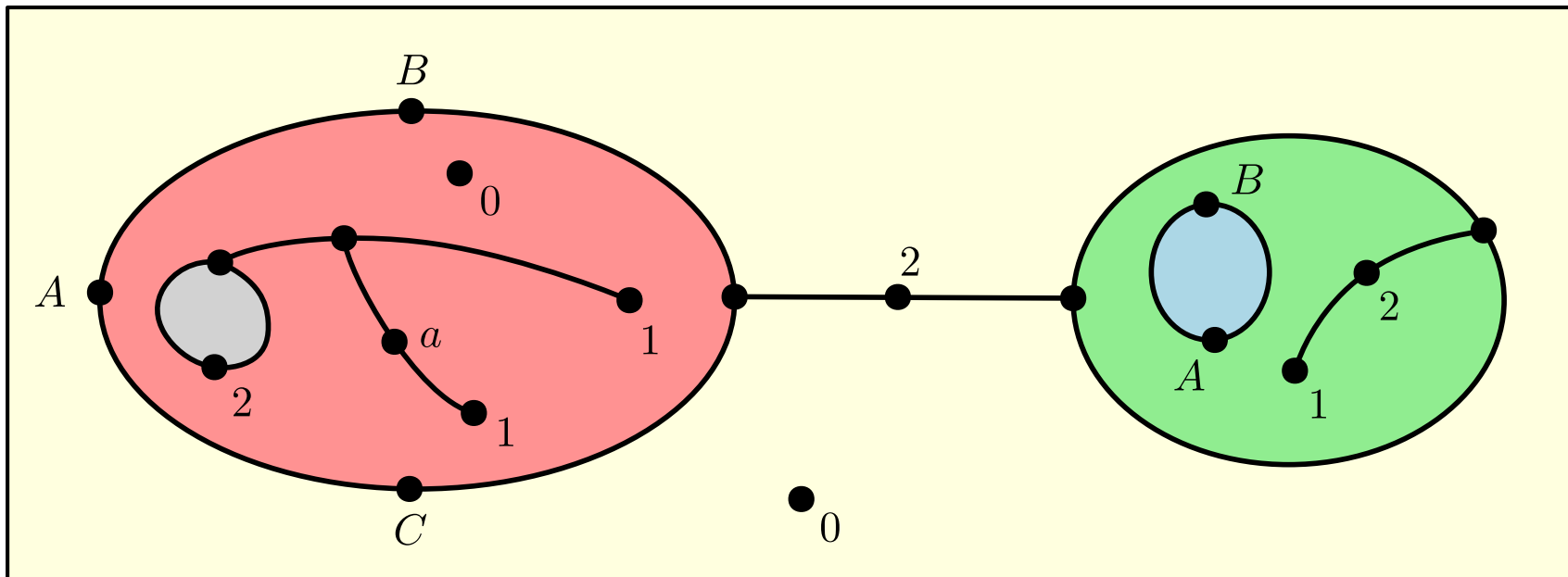
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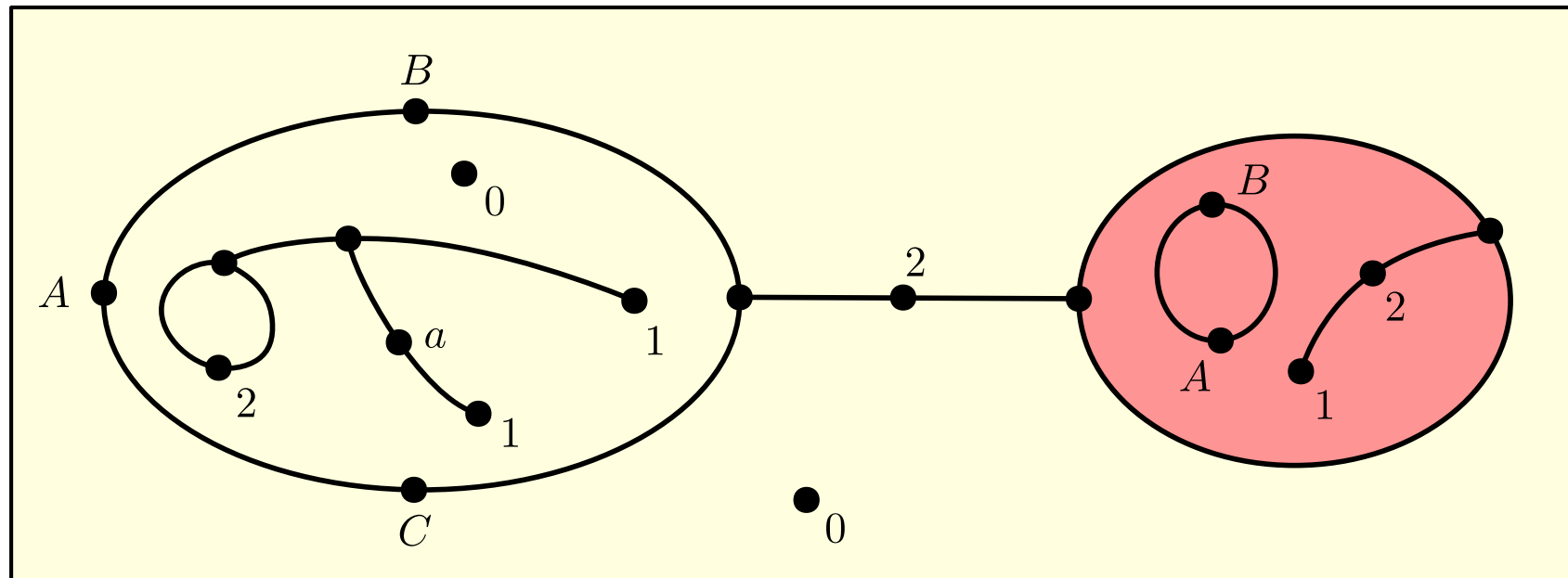
$\emptyset.12a1a.ABC \mid \emptyset.2ABC \mid 12.AB \mid AB$



Using the Nimber theory

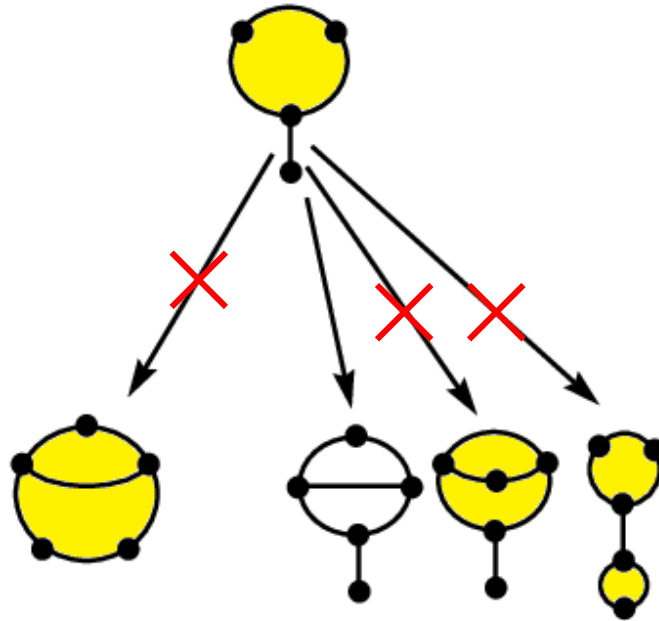
- Sprouts positions often consist of many independent parts that can be *analyzed separately* and whose results can be later merged together using *nimbers*.

$$0.12a1a.ABC \mid 0.2ABC + 12.AB \mid AB$$



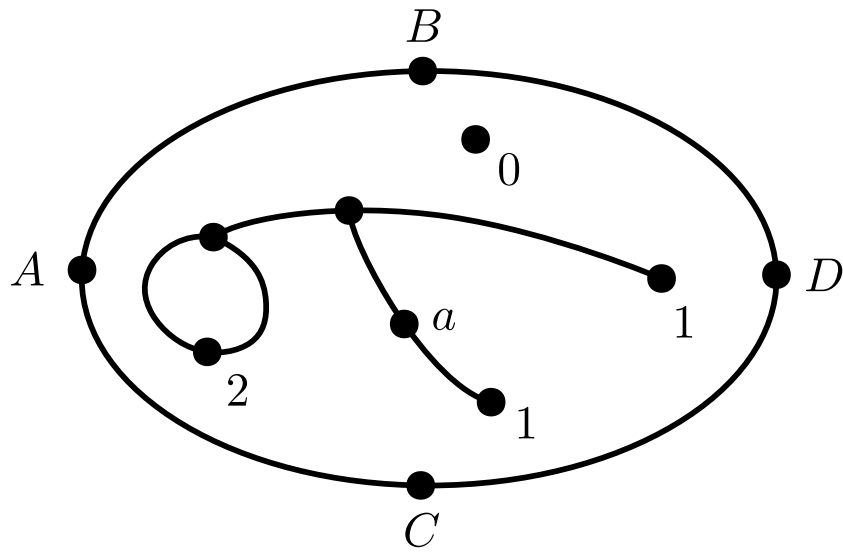
Training a perfect computer opponent

- We use a [pre-computed database](#) of positions.
- Some branches of the game tree do not have to be trained.



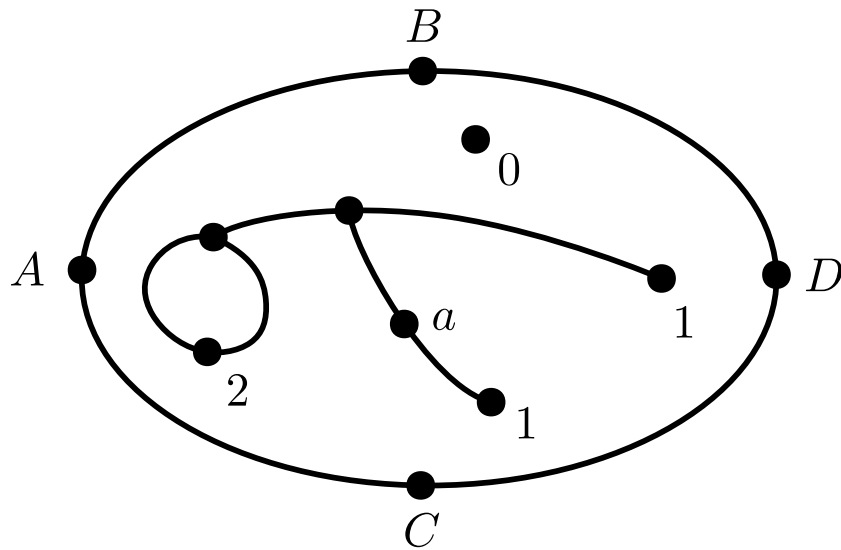
3rd barrier – drawing a computer move

$\emptyset.12a1a.ABCD|ABCD$



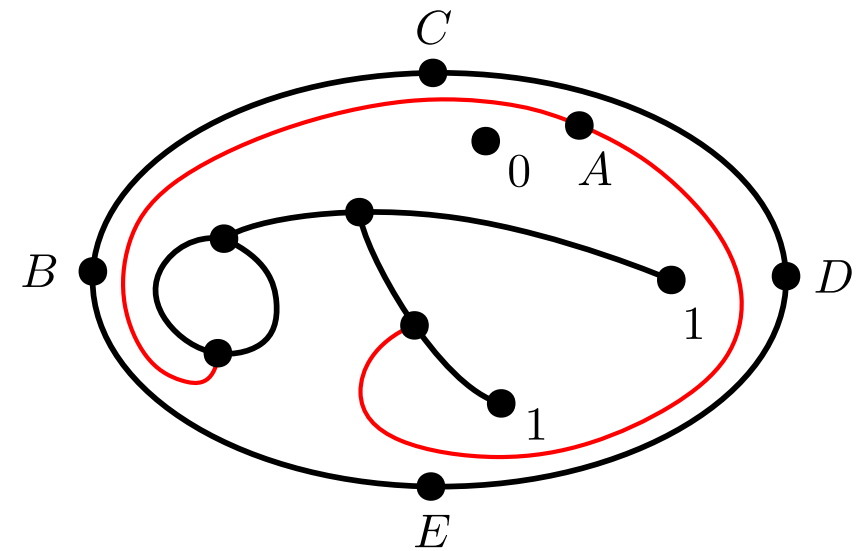
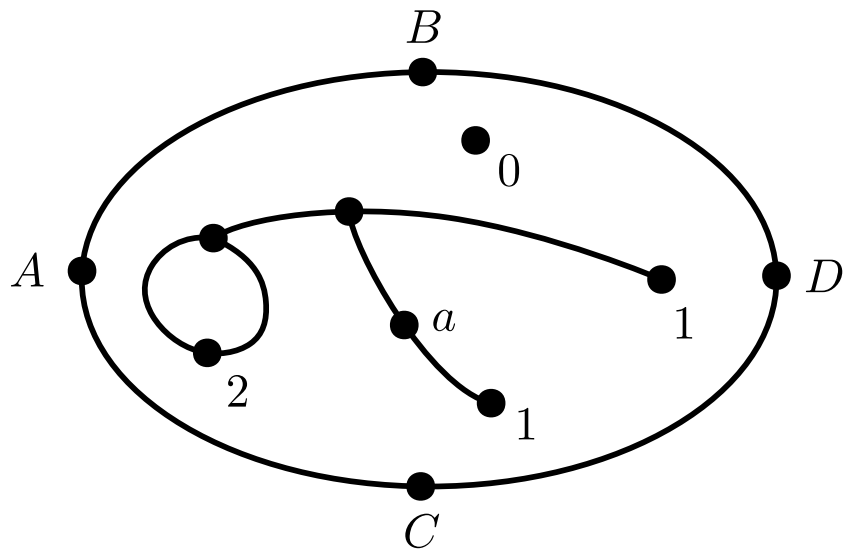
3rd barrier – drawing a computer move

$\emptyset.12a1a.ABCD|ABCD \xrightarrow{\text{a chosen move}} \emptyset.11A|A.BCDE|BCDE$



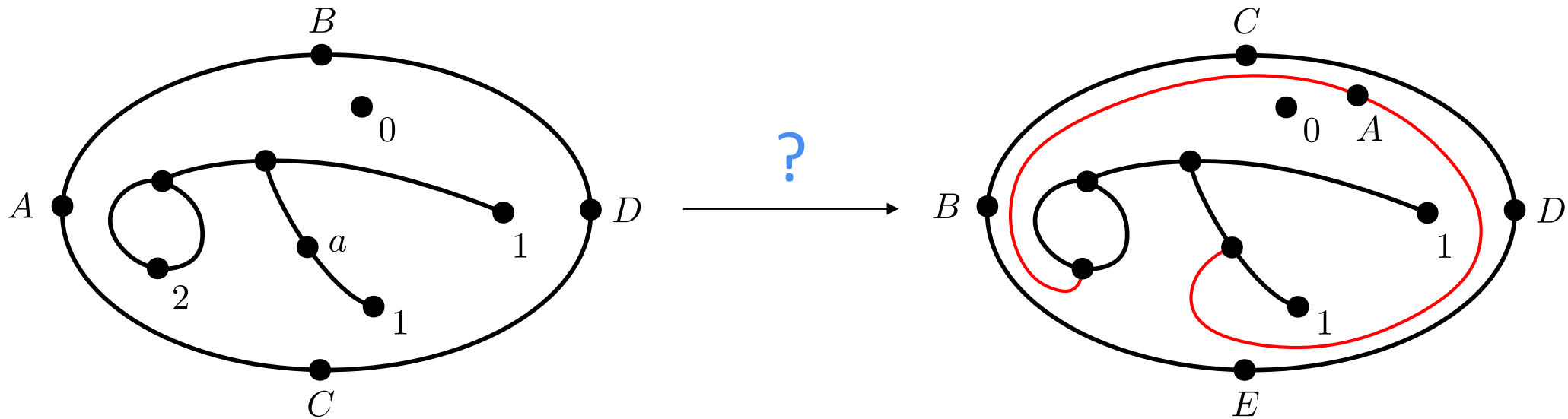
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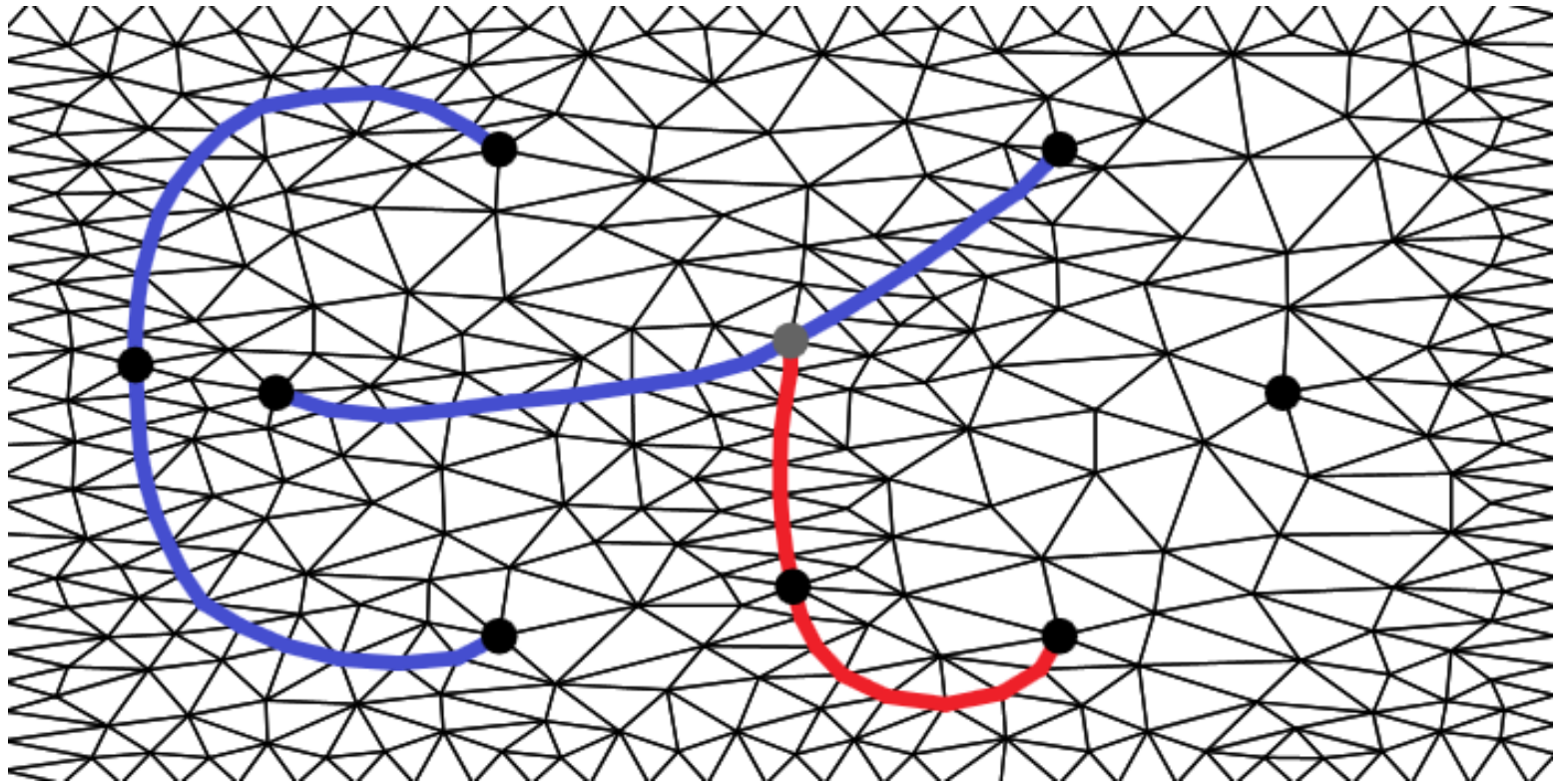


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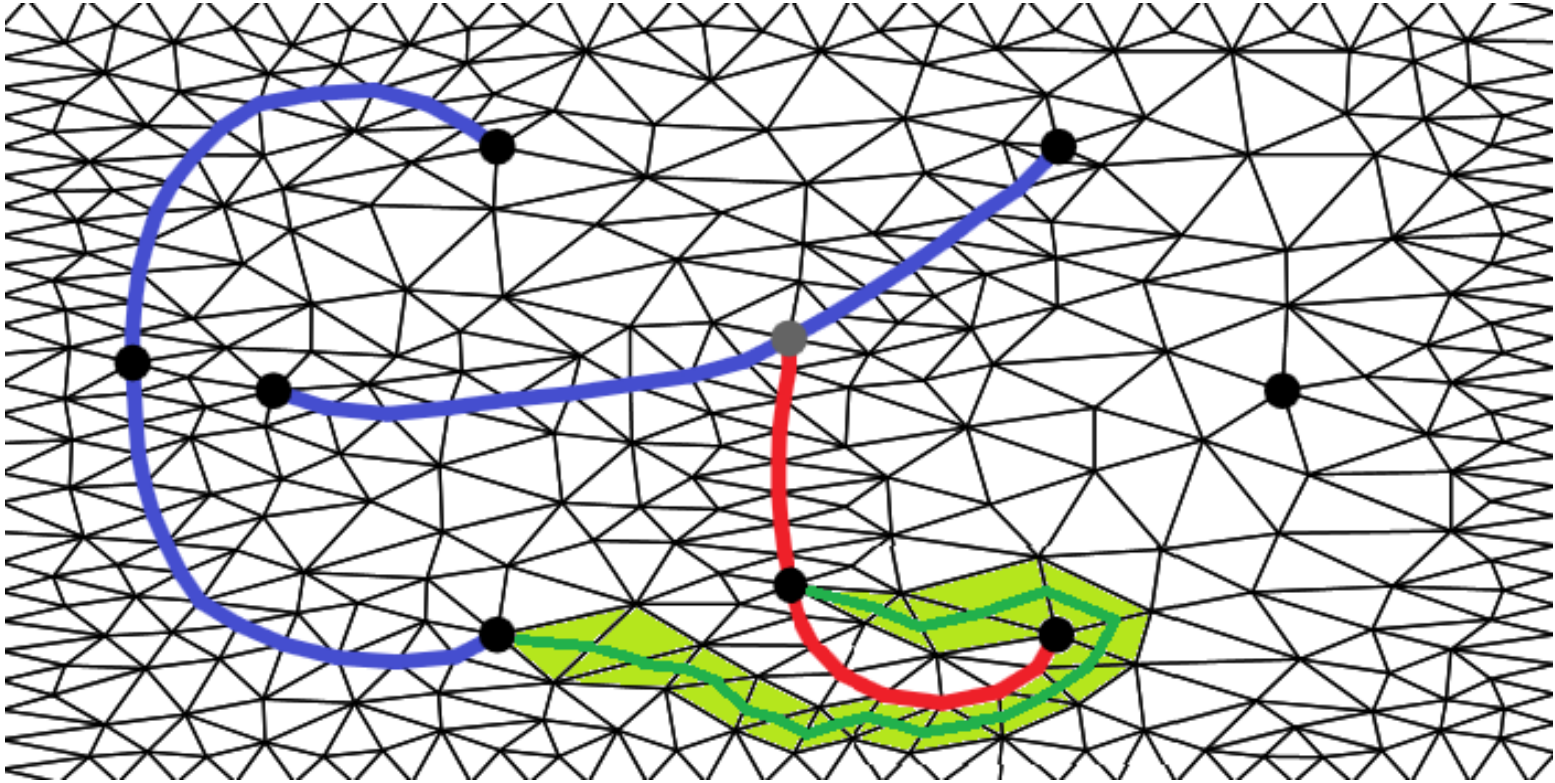
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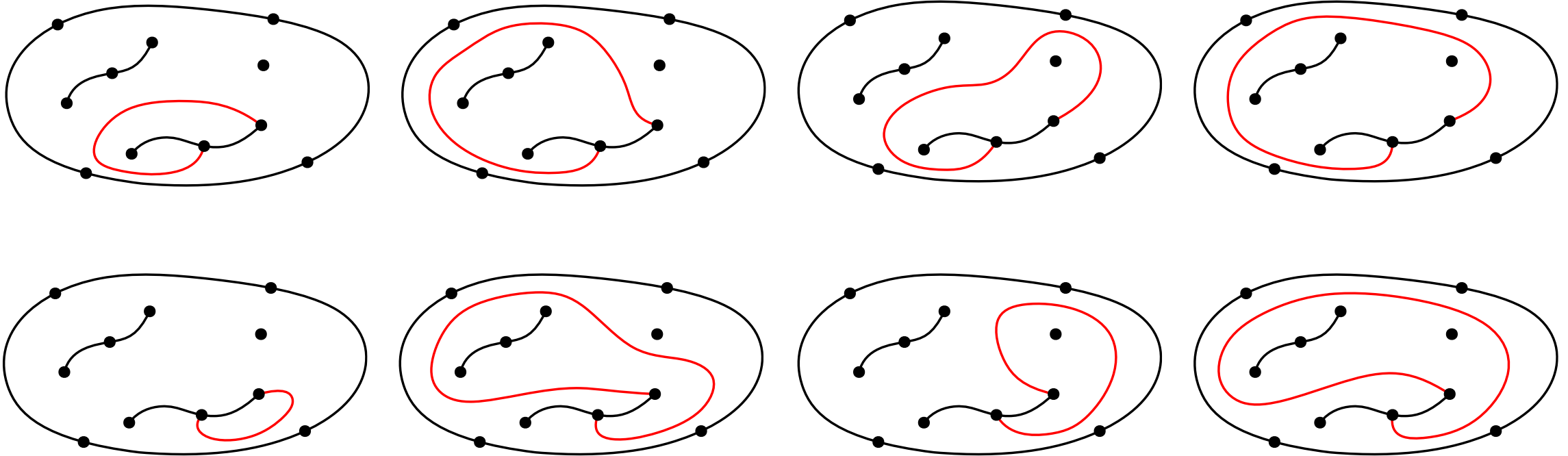
Triangulating a surrounding region



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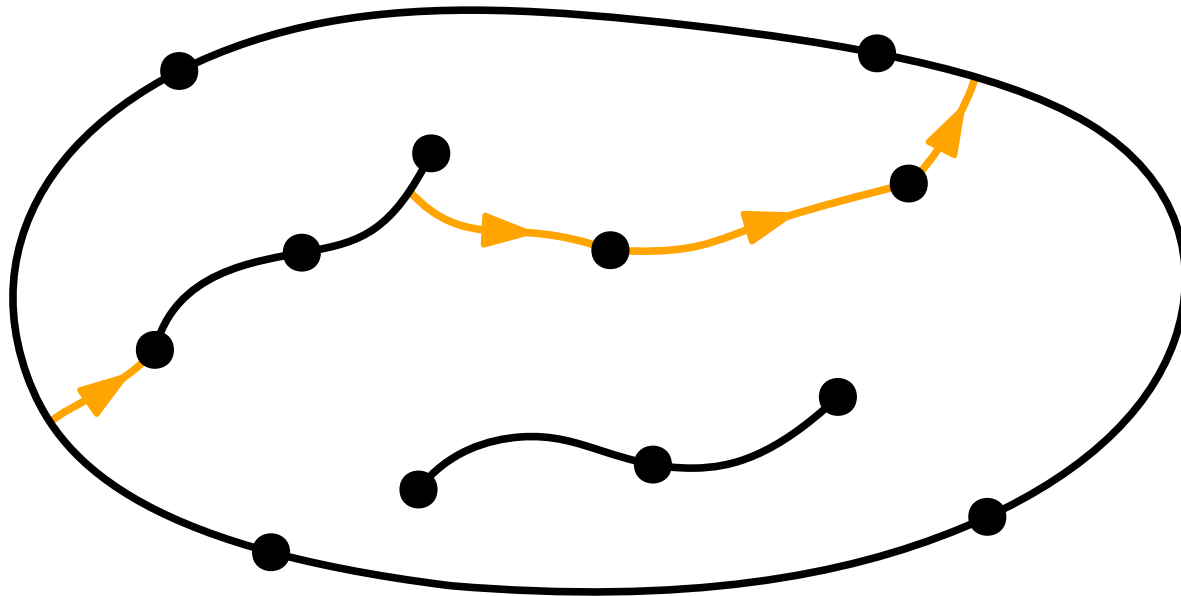


Problematic single-boundary moves



Spindle method

- We connect freely lying boundaries by an auxiliary curve called *spindle*.
- Each move then determines a unique intertwining of the spindle.

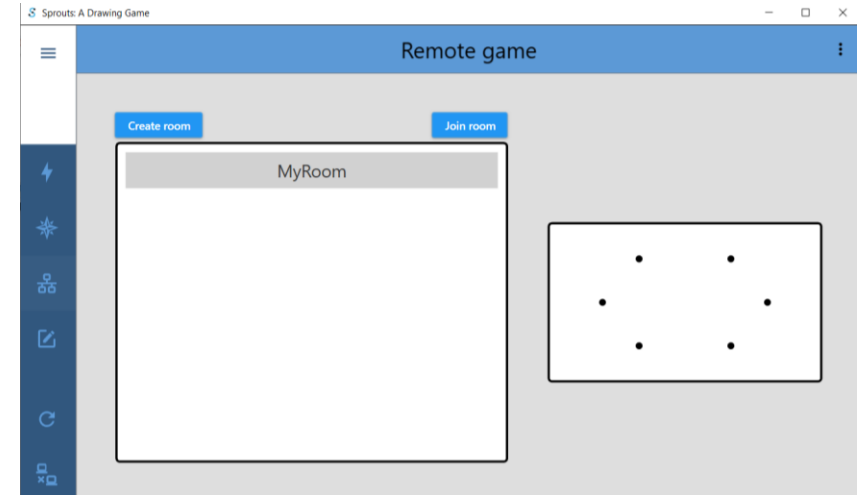
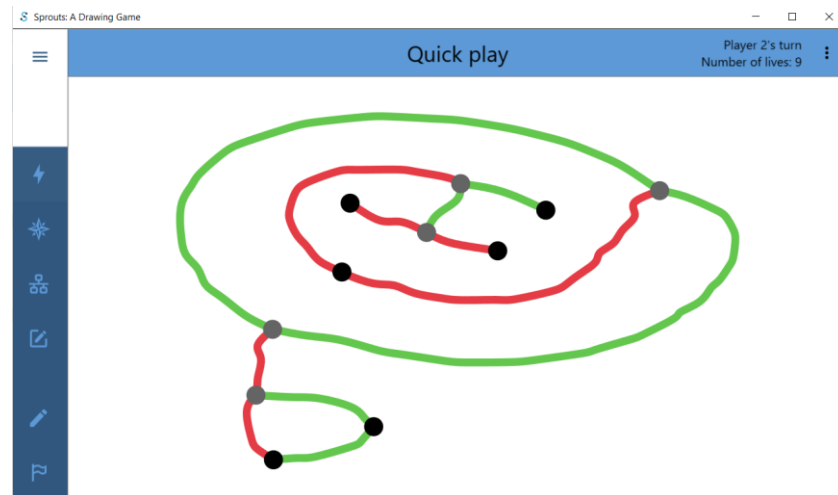
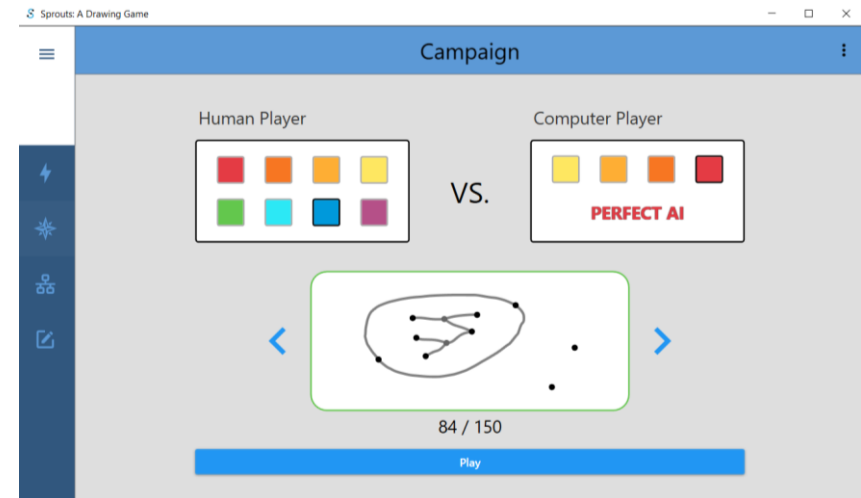


Conclusion

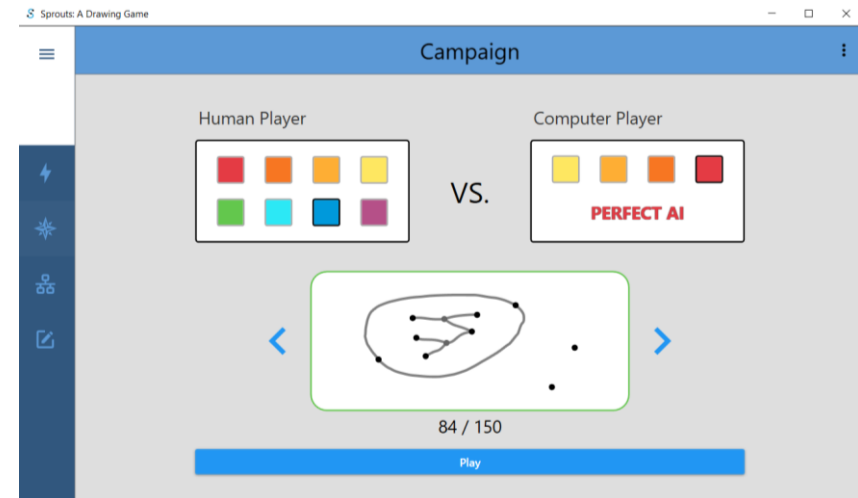
	Currently available	Free-form input	Crossings detection	Maintaining positions	Computer opponent	Remote game	Target platform
Sprouts - A Game of Maths!	✗	?	?	?	✓	?	iOS
SproutsPlus	✓	✗	✗	✗	✗	✓	iOS
Sprouts Game	✓	✓	✗	✗	✗	✗	iOS
UoU Sprouts Applet	✓	✓	✓	✗	✗	✗	Applet
3Graph	✓	✓	✓	✓	✓	✗	Windows
Sprouts: A Drawing Game	✓	✓	✓	✓	✓	✓	Windows

- We support games on up to 20 spots (*3Graph* [Reiss, 2009] only 8 spots) with a perfect AI on up to 11 spots (*3Graph* only 8).

Sprouts: A Drawing Game

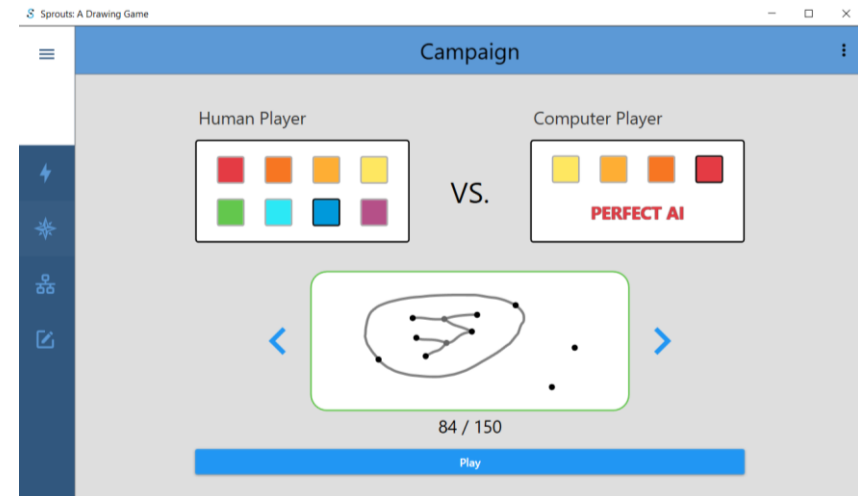


Sprouts: A Drawing Game



<https://kam.mff.cuni.cz/~cizek/Sprouts/>

Sprouts: A Drawing Game



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Thank you for your attention.